MANY-BODY PHYSICS AND SPONTANEOUS SYMMETRY BREAKING

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The idea of spontaneous symmetry breaking in many-body physics from personal perspective (Bose-gas, nuclear structure and a new approach of Generalized Density Matrix).

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1. Introduction

The notion of spontaneous symmetry breaking has quite general meaning, applicable both to physical phenomena and techniques used in theoretical physics. I start with short history of my personal experience with the problem.

My student years at Moscow State University (1946-1952) coincided with triumph of quantum electrodynamics. Curriculum of our new Physical-Technical Department required a part-time practice at a research institute. For me, it was the Institute nowadays named after Igor V. Kurchatov. After diffusion over some experimental laboratories, I found my place in the theoretical division. Our head, Arkady Migdal, was occupied by several problems, but for many years he had a favorite one, superconductivity. The new methods in QED, Green's functions, Dyson equations, Feynman graphs, were crying for adaptation to many-body problem. Naturally, Migdal with Victor Galitskii started such a work for Fermi systems. At first, my personal participation was only passive one, since I was deeply involved in plasma and beam physics working with Gersh Budker. Only after earning the doctorate (1955) I got free and decided to work on Bose system. At that time Migdal and Galitskii already found how to adjust the Green's function technique for many-body problems, having as their goal Fermi systems.[1] Very soon I realized that Bose systems are quite different.

2. Bose Gas

Peculiarity of a Bose system comes from a large, microscopic number of particles in the $\mathbf{p}=0$ state (*condensed phase*). This prevents direct use of Green's function technique. My idea was to split the system into two parts, the condensate, $\mathbf{p}=0$, and the rest, $\mathbf{p} \neq 0$, and consider them separately.

For the condensate one should introduce *n-body* Green's functions. Since they depend only on time variables and there is a small parameter, N^{-1}, one gets a hope. Accurate calculations turned out to be quite cumbersome. But the result was simple and physically transparent: to order $\sim N^{-1}$, the n-particle function has the form

$$iG_0(t_1...t_n;t'_1...t'_n) = K(t_1)...K(t_n)K^\dagger(t'_1)...K^\dagger(t'_n), \tag{1}$$

where

$$K(t) = \sqrt{n_0}\,\exp(-i\mu t), \tag{2}$$

and the chemical potential, $\mu = E_{N+1} - E_N \approx \partial E/\partial N$, can be calculated as $\mu = \sum(\text{loops})$, and the only new parameter is n_0, the density of the condensate.

The physics of eq. (1) can be understood from a particular case of a one-particle function in a non-interacting system, $iG_0^{(o)}(t-t') = \langle a_0(t)a_0^+(t')\rangle$, that describes the propagation of a particle from t' to t. In vacuum this process can proceed only by creating a particle at time t' and annihilating it at a later time t; in this case $G_0^{(o)}$ coincides with that in QED. But in the presence of N particles of the same type, the absorbed particle may be any one from the condensate. Time ordering of two events, creation at t' and absorption at t, is immaterial, within accuracy $\sim 1/N^{-1}$. That means the absence of correlation between creation and absorption processes, and the product form (1) corresponds to this physics. The use of (1) is equivalent to a simple ansatz: all operators a_0 and $a_0^\dagger$ should be replaced by

$$a_0(t) \Rightarrow \sqrt{n_0}\exp(-i\mu t), \quad a_0^\dagger(t) \Rightarrow \sqrt{n_0}\exp(i\mu t), \tag{3}$$

which is formally equivalent to a replacement of the quantum system, the condensate, by a uniform time-dependent field.

Exclusion of the condensate allows for direct application of QED techniques to the system of particles with $p \neq 0$, though with a new kind of elementary graphs and irreducible self-energy parts Σ_{ik} in Dyson-type equations. In addition to the ordinary Σ_{ik}, with an equal number of incoming and outgoing external lines ($i = k$), there are now self-energy parts Σ_{ik} where some external lines are replaced by the condensate "external field", and the numbers of incoming and outgoing lines may be different. For the single-particle Green's function, it is necessary to consider Σ_{11}, Σ_{20}, and Σ_{02}. The single-particle Green's function, disentangled from two coupled Dyson equations, was found as

$$G(p^0+\mu,\mathbf{p}) = \frac{p^0 + \varepsilon_\mathbf{p} + \Sigma_{11}^- - \mu}{\left[p^0 - (\Sigma_{11}^+ - \Sigma_{11}^-)/2\right]^2 - \left[\varepsilon_\mathbf{p} + (\Sigma_{11}^+ + \Sigma_{11}^-)/2 - \mu\right]^2 + \Sigma_{20}\Sigma_{02} + i\delta},$$

where $\Sigma_{11}^\pm = \Sigma_{11}(\pm p + \mu)$.

In the "gaseous" (low density) approximation, Σ_{ik} and μ were found as functions of the scattering amplitude $f(\mathbf{p}',\mathbf{p})$ and condensate density n_0. Quasiparticle excitation energy in the first order in $\sqrt{n_0 f_0^3}$ has the form ($m = 1$, $\hbar = 1$)

$$\epsilon(\mathbf{p}) = \sqrt{\left[\varepsilon_\mathbf{p} + 2n_0 f(\frac{\mathbf{p}}{2},\frac{\mathbf{p}}{2}) - n_0 f(00)\right]^2 - n_0^2\,|f(\mathbf{p},0)|^2}. \tag{4}$$

The second order approximation in $\sqrt{n_0 f_0^3}$ results in attenuation of quasiparticles (decay into two quasiparticles of lower energy). For small momentum it was found

$$\epsilon(\mathbf{p}) \approx p\sqrt{n_0 f_0}\left(1 + \frac{7}{6\pi^2}\sqrt{n_0 f_0^3}\right) - i\frac{3}{640\pi}\sqrt{n_0 f_0^3}\,\frac{p^5}{(n_0 f_0)^{3/2}}. \tag{5}$$

The Green's function allows one to find other physical quantities of interest, density and chemical potential:

$$n - n_0 \approx \frac{n_0}{3\pi^2}\sqrt{n_0 f_0^3}, \quad \mu = n_0 f_0\left(1 + \frac{5}{3\pi^2}\sqrt{n_0 f_0^3}\right). \tag{6}$$

Apart from the scattering amplitude, eqs. (5) and (6) contain only one parameter $n_0 f_0^3$, $f_0 = f(0,0)$. Just this parameter should be small for the validity of the gaseous approximation. It was not directly required that the total density n be small but eq. (6) shows that when n_0 is small, then n is also small. This means that it is not possible to reduce significantly the density of the condensed phase by increasing the interaction or the total density.

One general comment: solution of the problem becomes possible due to the replacement of the condensate, a quantum system, by a time-dependent external field due to the ansatz (3). Though quantitatively this operation is accurate up to the order $\sim N^{-1}$, it breaks the strong symmetry, total number conservation. Phonon-like excitations ($\epsilon(\mathbf{p}) \Rightarrow 0$ when $\mathbf{p} \Rightarrow 0$) emerge as a response to the symmetry breaking. This is a particular example of the general theorem on unavoidable appearance of such a mode (later on called *Goldstone mode*) after symmetry breaking.

One may look at the problem from a different point of view. The chemical potential μ is the only link between two subsystems (particles with $\mathbf{p} \neq 0$ and condensate). This is in full analogy with thermodynamic equilibrium of liquid with its vapor. Far from the phase transition, when fluctuations are small, properties of this vapor practically coincide with those of pure gas, but technically the gas is considered at fixed N, whereas the vapor at fixed μ.

Just as in the last example, one may from the beginning consider only the subsystem with $\mathbf{p} \neq 0$ at fixed μ, using the "corrected" Hamiltonian $\hat{H}' = \hat{H} - \mu\hat{N}'$ with $\hat{N}' = \sum a_{\mathbf{p}}^\dagger a_{\mathbf{p}}$, $\mathbf{p} \neq 0$, and operators a_0, $a_0^\dagger$ replaced by a c-number $\sqrt{n_0}$. The Hamiltonian $\hat{H}'$ allows for the use of the QED techniques. Two free parameters, n_0 and μ, can be then found using $\langle n' \rangle = n - n_0$ and minimization of ground state energy. Such an approach was demonstrated by Hugenholtz and Pines, Ref. 2.

Two papers with the results on Bose systems were sent for publication in summer 1957.[3,4] Before that, I gave a talk at the Landau weekly seminar after a preliminary personal discussion with him. (This was the usual procedure for all to-be-speakers at the Landau seminar). It was the time when the BCS theory of superconductivity became known in Moscow and excited the physical community. Naturally, I start to think on possible application of my results to superconductivity. But external circumstances made a different decision for myself. Director of the institute, I.V. Kurchatov, decided (and made it possible) to resume collaboration with European

research centers. So, two young physicists, Vilen Strutinsky and myself, were sent for a year to the Niels Bohr institute in Copenhagen. (A few month later Victor Galitskii was sent to Rudolph Peierls in Birmingham).

3. 1957-58, Niels Bohr Institute

The main activity, research and discussions there were around the *unified nuclear model* by Aage Bohr and Ben Mottelson. The general scheme was clear and well confirmed. The base is a spherical mean field with shell structure and its evolution with deformation. It allows one to establish quantum numbers of odd nuclei and single-particle excitations for spherical and deformed nuclei. The main interest was for collective excitations, vibrations in spherical nuclei, and vibrations + rotation in deformed ones, since their properties, calculated with different residual interactions, did not agree with experiments. Among the puzzling theoretical results, in particular, were predictions for inertial parameters for vibrations and rotation.

After the BCS theory of superconductivity, Bohr, Mottelson and Pines made a suggestion[5] that the energy gap found in the spectra of even-even nuclei may be caused by correlation effects similar to those in superconductors. It was extensively discussed, with the use of the *seniority* quantum number, what is the analogue of a Cooper pair in the shell model (say, in a single j-level), how pairs occupy the shell etc. I decided to enter the problem from another, many-body, direction, thinking, as was later formulated in my article 6, that

... such correlations may also effect other nuclear properties, which have no analogue in superconductors, connected with the finite size of nuclei and the shell structure of the single particle levels.

In particular, I bore in mind long standing and puzzling theoretical results (obtained in fact for any reasonable interaction):

- Spherical shape is stable only for closed shell nuclei. Even one-two valence particles result in the deformation that gradually increases with growing occupation.
- The inertial parameter for vibrations is of hydrodynamic value (too small).
- The rotational moment of inertia is close to the rigid body value (too large).

Results of my attempts may be understood from *Concluding Remarks* in Ref. 6:

Starting from basic assumption that a pairing correlation of a "superconducting" type exists between nucleons, we have attempted to investigate consistently the effects of this correlation in different nuclear phenomena. Although the calculations are based on a rather idealized model, a great number of experimental facts of a different kind are explained in a natural way from a single point of view, viz.,

(a) Stability of the spherical shape of nuclei near the closed shells;

(b) Sharp transition between spherical and deformed nuclei;

(c) Significant reduction of the moment of inertia from the value for rigid rotation;

(d) Existence of low-energy vibrations in spherical nuclei near the bound of instability.

A few comments in addition. The pairing correlations were treated as in the BCS

theory with particle number non-conservation (*μ-system* instead of *N-system*). The finite size of the nuclear mean field breaks also translational symmetry; a deformed mean field breaks rotational symmetry. Collective low energy vibrations (inside the gap) and rotations are just the response to this symmetry breaking, that is Goldstone modes. The problem of rotation has its own peculiarity that until now stimulates new theoretical approaches. First, let us cast a glance on existing many-body techniques.

4. Microscopic Many-Body Theory: A General View

In practice, the starting point and the base is always a mean field (MF) and the view on the system as composed of independent quasiparticles (QP). The MF is supposed to be, at least in principle, self-consistently determined from the same QP. At the next stage one can explore symmetry and dynamic properties of the MF, say, by its response to an external field, and recognize the structure of collective excitations. An external field for exploring the response of the system may be rather specific. One example, considered above, is the condensate in Bose system, acting on the subsystem with $\mathbf{p} \neq 0$. The second example, rotation induced by an external torque (*cranking*), will be considered below.

The Hartree-Fock (HF) approach to the self-consistent calculation of MF can be formulated as an equation for the density matrix $\rho_{12} = \langle 0|a_1^\dagger a_2|0\rangle$,

$$[h(\rho), \rho] = 0; \quad \rho^2 = \rho; \quad h(\rho_a) = \varepsilon_a + \mathrm{Tr}_b\,(V_{ab}\rho_b), \tag{7}$$

where the mean-field Hamiltonian $h(\rho)$ contains kinetic energy ε and the self-consistent potential, generated by the interaction V_{ab}. The HF method can be formulated also as a variational problem with fixed N.

The Hartree-Fock-Bogoliubov (HFB) method is an extension of HF, which includes Cooper pairing, when the N-violating pair transfer matrix elements $\langle N-2|\Delta|N\rangle$ and $\langle N+2|\Delta^\dagger|N\rangle$ are large. In variational form one looks for $E_0(\mu) = \min\langle \hat{H} - \mu\hat{N}\rangle$ with μ as a Lagrange multiplier to be determined from $\langle\hat{N}\rangle = N$ or $\partial E_0/\partial\mu = -N$. In terms of statistical thermodynamics, the transition from HF to HFB means replacement of the system under investigation. The system with a fixed number of particles and Hamiltonian H is replaced by the system with a fixed chemical potential, μ, with the grand canonical potential $H - \mu N$. Naturally, the solution for the μ-system gives only average quantities over few neighboring N-systems. Usually such an average description is satisfactory, but for nuclei near the spherical-deformed transition and weakly bound nuclei the pairing should be treated more accurately.[7]

Rotation seems to allow the same approach as pairing. In this case there are large matrix elements of the quadrupole moment $\langle J|Q_{2\lambda}|J'\rangle$. Therefore it seems possible to look for $E(\Omega) = \min\langle\hat{H} - \Omega\hat{J}\rangle$, with a Lagrange multiplier, angular velocity Ω, to be found from $\langle\hat{J}\rangle = J$, or $\partial E_0/\partial\Omega = -J$. In spite of formal similarity to the pairing case, rotation is much more complicated, since the angular momentum is

the vector with non-commuting components. Usually only one projection is used assuming plane rotation of a nucleus with axially symmetric deformation, which allows one to find one moment of inertia. This may be reasonable for even-even nuclei when adiabatic rotation does not excite intrinsic degrees of freedom. But nuclear rotation turns out to be a phenomenon much more diverse and rich.

5. Nuclear Rotation

During last two-three decades, a lot of experimental data were collected mostly with large γ-ray arrays. Hundreds of new collective bands were found and precisely measured up to very high angular momenta. Among the new discoveries are *superdeformation*[8] (with axis ratio $\sim 2:1$), *identical bands*[9] (which coincide in neighboring nuclei with accuracy 0.1%), *bifurcation*[10] (small splitting at some value of I of a band with spins $I = 2n$ into two sub-bands with $I = 4n$ and $I = 4n + 2$), *backbending*[11] (transition from a band with pure rotational I to a band based on a quasiparticle pair with high intrinsic angular momentum), *magnetic rotation*[12] (with strong $M1$ transition matrix elements), *wobbling motion*[13] etc.

How adequate are theoretical tools for the richness of experimental data? One should distinguish between phenomenological and microscopic approaches. From symmetry and specific features of a rotational band, one can guess on the structure of a *phenomenological Hamiltonian* for its description, e.g. of the form

$$H = AI^2 + BI^4 + C(I_+^2 + I_-^2) + D(I_+^4 + I_-^4), \tag{8}$$

where the coefficients are to be chosen to fit experimental data, say near a bifurcation point. This might be reasonable provided the coefficients are determined consistently from intrinsic nuclear structure. Otherwise, it is a pure artificial problem of a rotor with a specific Hamiltonian.

Particle-rotor model (PRM))[14] is an attempt to describe the interaction between the rotor and an odd particle in the form

$$H_{\mathrm{PRM}} = \frac{1}{2\mathcal{J}}(\mathbf{I} - \mathbf{j})^2. \tag{9}$$

This model permits one to analyze interplay between competing alignment forces, deformation and Coriolis, acting on a particle. But the restriction to Coriolis, a pure kinematical interaction with an unidentified rotor, does not allow for using the PRM for quantitative description of rotational spectra in odd nuclei.

The *cranking model* (CM) is, in fact, the only practical tool for microscopic calculations of rotational phenomena. One starts with a many-body Hamiltonian with an external torque,

$$\hat{H}(\mathbf{\Omega}) = \hat{H} - \mathbf{\Omega}\hat{\mathbf{J}}, \tag{10}$$

and proceeds with HF or HFB. This results in the *shell model in the rotational frame*, where the MF and single-particle levels are functions of angular velocity $\mathbf{\Omega}$, that is supposed to be excluded from final results and replaced by the angular

momentum. The problem is that such a replacement cannot be done uniquely. In the traditional CM[15] one assumes the rotational axis coincide with one of the principal axis of the deformed field (*principal axis cranking*, PAC). For complicated rotational bands, for example, in odd-odd nuclei, the direction of the rotational axis is a subject for adjustment to fit experimental spectra (*tilted axis cranking*, TAC)[16]. Defects of CM are evident: it violates rotational symmetry, does not provide a regular way to estimate and correct the errors and prevents direct application to the problems of interaction between rotational and intrinsic excitations. "Average over J" description, imminent in the CM, becomes too crude for the cases where an interplay between rotation and intrinsic excitations results in an abrupt change in the band structure, an analogue of a phase transition.

6. Interim Summary

- Sacrifice of strong symmetry opens way for simple theoretical description of spontaneous symmetry breaking. The symmetry is controlled *in average* by Lagrange multipliers, which have a definite physical meaning. This can be interpreted as a replacement of the system under investigation.
- The method is satisfactory for macroscopic systems, where an average statistical description is the conventional one.
- In mesoscopic finite systems the description "in average" gives a crude approximation, especially in cases with sharp structural changes.
- A striking example is given by diversity of nuclear rotation, where many rotational bands demonstrate sharp structural changes.
- Microscopic methods with symmetry conservation are evidently needed.

A basic microscopic approach was suggested by Kerman and Klein[17] and developed by Klein with collaborators.[18] The starting point is equations of motion (EoM) for the matrix elements of single-particle creation and annihilation operators with precise conservation of angular momentum and particle number. Unfortunately, even after many years of reformulation, the method is still too complicated to serve as a practical tool. A similar, *generalized density matrix* (GDM) approach, based on EoM for bilinear combinations $aa^\dagger$ was suggested by Zelevinsky and myself.[19] Solutions obtained for several problems[20,21,22] demonstrated its practical efficiency. Since the method is not familiar to wider theoretical community and seems to be underestimated, I try to explain main ideas of the GDM approach.

7. Generalized Density Matrix (GDM)

Systems with spontaneous symmetry breaking always have a set of states (Goldstone modes) $|\Lambda\rangle$ strongly linked by large matrix elements (as $\langle N-2|\Delta|N\rangle$ for pairing and $\langle J|Q_{2\lambda}|J'\rangle$ for rotation). Therefore, we need a method which can handle matrix elements $\langle\Lambda|a_1 a^\dagger_{1'}|\Lambda'\rangle$, diagonal and off-diagonal, on the same footing as expectation values for the ground state in the HF method. The first step in this direction

is known as the random phase approximation (RPA), when, for small amplitude vibrations, we assume for off-diagonal matrix elements

$$\langle 0|a_1 a^\dagger_{1'}|\Lambda\rangle \ll \langle 0|a_1 a^\dagger_{1'}|0\rangle \approx \langle \Lambda|a_1 a^\dagger_{1'}|\Lambda\rangle).$$

It results in the equation for the *transition density matrix*, $\langle 0|a_1 a^\dagger_{1'}|\Lambda\rangle$.

The GDM offers a possibility to go beyond mean field and RPA. The basic assumption is the possibility to separate from the Hilbert space of the many-body system a restricted *collective band* of states strongly linked by dynamics among themselves but only weakly with the states outside the band. In other words, dynamics may be saturated inside the collective band; in this language the HF method means the saturation of the dynamics by the ground state. The outcome is the nonlinear equation of the form

$$[H + h(R),\, R] = 0, \tag{11}$$

where $h(R)$ has structure similar to (7). All quantities in (11) are matrices in *combined* space, the product of single-particle ("intrinsic"), $|1\rangle$, and collective, $|\Lambda\rangle$, spaces. To emphasize this fact, we introduce, for all quantities in combined space, a convenient notation

$$\langle \Lambda|R(11')|\Lambda'\rangle \Rightarrow \langle \Lambda 1|R|1'\Lambda'\rangle.$$

Here the GDM operator R is a bilinear combination of creation and annihilation operators. If pairing is of importance, it is useful to consider the GDM in the "spinor" representation,

$$\hat{R}(1,2) = \begin{pmatrix} a_1 a_2^+ & a_1 a_{\bar{2}} \\ a_{\bar{1}}^+ a_2^+ & a_{\bar{1}}^+ a_{\bar{2}} \end{pmatrix}, \tag{12}$$

where tilde means time-conjugation. The Hamiltonian $H \Rightarrow \langle \Lambda|\mathcal{H}|\Lambda'\rangle$ depends, naturally, only on collective quantum numbers Λ, and is reduced to a diagonal matrix when $|\Lambda\rangle$ are eigenvectors of $\mathcal{H}$.

Equation (11) is of the HF type but in combined space, where $H + h(R)$ plays the role of the self-consistent MF Hamiltonian with the single-particle part $h(R)$ and collective part H. Coupling between single-particle and collective degrees of freedom reveals in dependence of $h(R)$ not only on single-particle variables but also on collective ones. Like in ordinary HF theory, eq. (11), should be supplied with a normalization condition for the GDM, $R^2 = R$. The GDM $\langle \Lambda 1|R|1'\Lambda'\rangle$ has a simple physical meaning: the trace (qR) over intrinsic space with a one-body operator (q), say quadrupole moment q_{ik}, results in the operator of the same type in collective space (Q_{ik}).

7.1. *Nuclear rotation revisited*

Select, as an example, the collective band as an isolated rotational band of a well deformed nucleus. At the next stage we can use the symmetry of the band governed by the rotational group $SU(2)$. The states within the band are uniquely labelled

by the total spin J and its projection J_z onto the laboratory-fixed axis z, dynamic variables are Euler angles. We outline several steps which lead to solution the problem.

(a) Angular momentum conservation in combined space can be written as

$$[\mathbf{J}+\mathbf{j},R]=0; \quad [\mathbf{J}+\mathbf{j},h(R)]=0. \tag{13}$$

(Simultaneous rotation in both intrinsic and collective spaces cannot change R).

(b) There exists a transformation D, unitary in the intrinsic space and acting as a "shift" in the collective space,

$$D^{-1}(J_i+j_i)D=J_i. \tag{14}$$

For the transformed quantities $D^{-1}RD=R'$ and $D^{-1}hD=h'$, one has

$$[\mathbf{J},R']=0; \quad [\mathbf{J},h']=0,$$

which means that R' and h' do not depend on Euler angles, and the only possible dependence on collective variables is that on I_α, angular momentum projections on body-fixed axes. The collective Hamiltonian, on the other hand, is a function of components I_α transformed as

$$D^{-1}I_\alpha D=I_\alpha-j_\alpha, \tag{15}$$

so that, after the D-transformation, the collective Hamiltonian H depends also on intrinsic angular momentum.

(c) Due to rotational invariance of ε and the two-body interaction, one has $D^{-1}\varepsilon D=\varepsilon$; $D_a^{-1}D_b^{-1}V_{ab}D_bD_a=V_{ab}$, so that

$$h'(R)=D^{-1}hD=h(R'), \tag{16}$$

and the GDM equation after the D-transformation takes the form

$$[H(I_\alpha-j_\alpha)+h(R'),\,R']=0. \tag{17}$$

7.2. *Solution of the equation*

We start with a selection of the operator form of the collective Hamiltonian H. For instance, with the help of theoretical vision and/or experimental data we assume H in the form of expansion

$$H(I)=H_0+\frac{1}{2}H_{ik}I_iI_k+\frac{1}{4}H_{iklm}I_iI_kI_lI_m+\ldots \tag{18}$$

with coefficients to be found.

(ii). We look for the solution of (17) as an expansion

$$R'=R_0+R_i(I_i-j_i)+R_{ik}(I_i-j_i)(I_k-j_k)+\ldots. \tag{19}$$

With the help of eq. (16), it is easy to see that $h(R')$ is expanded as

$$h(R')=h_0+h_i(I_i-j_i)+h_{ik}(I_i-j_i)(I_k-j_k)+\ldots \tag{20}$$

with the coefficients

$$h_0 = \varepsilon + \mathrm{Tr}\,(VR_0) \equiv \varepsilon + w(R_0); \quad h_i = w(R_i); \quad h_{ik} = w(R_{ik}); \ldots \tag{21}$$

The main equation (17) with the ansatz of (20) and (21) is split into a set of equations according to the powers in $(I_i - j_i)$. The scalar equation for R_0 is nonlinear being of the HF type; the vector equation for R_i and higher tensor equations are linear equations of the RPA type with right hand sides found at the previous steps. Solutions obtained for R_i, R_{ik}, ... depend on the parameters H_{ik}, H_{iklm}, ., which should be established from a consistency condition.

7.3. *Consistency condition*

In the basic equation (11), the collective Hamiltonian was defined as a matrix $H = \langle\Lambda|\mathcal{H}|\Lambda'\rangle$, that is, as a "projection", mapping of the many-body microscopic Hamiltonian $\mathcal{H}$ onto the collective band. In terms of the density matrix R, this condition can be written as

$$H \Leftrightarrow \mathrm{Tr}\,(\varepsilon R) + \frac{1}{2}\mathrm{Tr}_a\,\mathrm{Tr}_b\,(R_a V_{ab} R_b). \tag{22}$$

Both sides in (22) are given by expansion in powers of I_i. Equating the coefficients in the expansions, we shall obtain numerical parameters H_{ik}, H_{iklm}, ..., as traces over intrinsic states of the nucleus. At this stage we get a macroscopic phenomenological Hamiltonian with its parameters consistently defined from microscopic theory. It is possible to use other consistency conditions, in particular for the total angular momentum, $J_i = \mathrm{Tr}\,(j_i R)$.

7.4. *Applications*

The GDM techniques were successfully applied to several problems.

7.4.1. *Pairing*

This is the simplest example of broken symmetry. The collective band contains the set of the ground states with different particle numbers, $\ldots N-2, N, N+2, \ldots$, and N plays the role of conserved momentum. Its intrinsic counterpart is the Pauli matrix τ^3 in the spinor representation (12). The solution of the problem gives the "moment of inertia" for *pair rotation*, that is pair separation energies.

7.4.2. *Translational motion*

In this case the GDM gives a prescription for transformation of the single-particle density matrix ρ (for example, the HF solution) into the GDM R with the conserved linear momentum $\mathbf{P}$. The transformation is given by

$$R = \exp(-i\mathbf{pX})\exp(i(\mu/M)\mathbf{Px})\rho\exp(-i(\mu/M)\mathbf{Px})\exp(i\mathbf{pX}), \tag{23}$$

where $\mathbf{x}$ and $\mathbf{X}$ are the particle and center-of-mass coordinates, respectively, M is the total mass and μ is the reduced mass of the particle with respect to the rest of the nucleus. The self-consistent solution (23) gives the total momentum $\mathbf{P} = \mathrm{Tr}\,(\mathbf{p}R)$ and particle number $A = \mathrm{Tr}\,R = M/m$, where m is the nucleon mass. This exact solution is obtained for the case without pairing and velocity dependent forces. The case with pairing can be solved with accuracy A^{-1}.

7.4.3. *Low spin rotation*

In the case of even-even nuclei considered above as an example, the GDM determines the moments of inertia (all three for triaxial deformation). For axial symmetry, the result for the moment of inertia in the adiabatic approximation coincides with that of the cranking model including the term, caused by the rotational distortion of the MF. Non-adiabatic corrections have improved the agreement with experiments.

7.4.4. *High spin rotation*

In the asymptotic high spin region, $I \gg j$, eq. (11) can be written in the form

$$[h(R') - \mathbf{\Omega j},\, R'] = i(\mathbf{I} \times \mathbf{\Omega})\frac{\partial R'}{\partial \mathbf{I}}, \tag{24}$$

where $\mathbf{\Omega} = \partial H(\mathbf{I})/\partial \mathbf{I}$, and $\mathbf{I}$ should be treated as a classical quantity. This is, in fact, *time-dependent HF cranking*, since the right hand side, due to the Euler equation $\dot{\mathbf{I}}(t) = \mathbf{I}(t) \times \mathbf{\Omega}(t)$, can be written as $i\dot{R}(t)$.

7.4.5. *Interplay of rotation with intrinsic excitations*

In this case the collective Hamiltonian H in (11) depends not only on angular momentum but also on additional quantum numbers, which characterize other degrees of freedom, for example, vibrations or valence particle(s). The "collective band" in this case is a set of rotational bands based on different band heads, whose characteristics can be chosen as additional quantum numbers of the collective band. The interacting bands built on the RPA β- and γ-phonons were successfully considered.[23]

8. Conclusion and Outlook

The most interesting and promising GDM applications are still waiting for more extensive exploration (collective Hamiltonian for very soft nuclei or other mesoscopic systems with ill-defined shape; enhancement of weak interactions; the limit of many chaotically interacting collective states in relation to random matrix theory etc.) I hope the new results will appear in the near future.

Spontaneous symmetry breaking is a widespread phenomenon in different fields of physics, from condensed matter to elementary particles. In the latter field, for example, a fundamental chiral symmetry breaking is supposed to be restored at

the phase transition from hadron matter to quark-gluon plasma. Methods of many-body theory, being developed for one field of physics, are often successfully used in seemingly remote fields; the application of the methods of QED for condensed matter physics was mentioned above.

The present conference gives a marvelous chance for exchange of results, findings and critics as well. That is why I have included in my talk the Generalized Density Matrix approach. It was proved to be successful in nuclear physics. Let me hope that it finds its place in other fields.

To conclude, I would like to express my gratitude to the Feenberg Medal committee for the award and exciting opportunity to take part in this meeting.

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