EFFECT OF PAIRING CORRELATIONS ON COLLECTIVE NUCLEAR PROPERTIES

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SUMMARY

A method analogous to that developed in the new theory of superconductivity is applied to nuclei in order to investigate the influence of the coherent pairing interaction on various nuclear properties. We consider, in particular, the effect on the nuclear collective properties. For strongly deformed nuclei the moment of inertia is rather sensitive to the effect of pairing correlations and is found to be reduced from the value for rigid rotation by a factor of the order of that observed. For nuclei in regions near closed shells the pairing correlations give rise to a spherical equilibrium shape and low energy vibrational modes of excitations. The vibrational frequencies and inertial parameters obtained from the present model are in qualitative agreement with observed trends.

After the appearance of the new theory of superconductivity [1, 3] it was suggested [4] that correlations of similar type as those considered for the electron system in superconductors may play an important role in the nuclear excitation spectrum. We have considered a systematic extension of the methods applied in superconductivity to treat the nuclear system. For the latter system it is necessary to take into account the finite size effects, in particular the shell structure of the single particle levels.

Using the methods developed by N. N. BOGOLYUBOV [2] we consider the pairing correlation between two nucleons in states of opposite angular momentum projections. The correlation effect is taken into account by means of a canonical transformation from the original interacting nucleons to new independent quasiparticles—the elementary excitations. The ground state of the

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system in terms of the new quasiparticles is the "vacuum" state. The essential part of the pairing interaction comes into the "vacuum" energy and into the intrinsic structure of the quasiparticles. Therefore, even if the residual interaction between the quasiparticles is neglected one may investigate the influence of the pairing interaction on various nuclear properties.

We consider a system of nucleons moving in an axially symmetric self-consistent field. The single particle levels in this field are assumed to be divided into groups (shells), which in the limit of zero deformation become highly degenerate levels. The energy of the state ν^+ and ν^-, which differ only in the sign of the angular momentum projection, is equal to ε_ν. The matrix of the interparticle interaction $G(r_1 - r_2)$ of the form $\langle \nu^+\nu^- | G | \nu'^-\nu'^+ \rangle$ is assumed to be constant ($= G$) for the transitions inside the shell. Transitions between the shells cause only a certain renormalization of the interaction and therefore each shell can be treated independently. The shell is characterized (*) by boundary levels ε' and ε'', by the total number of pair states Ω and by the occupation factor $\chi = 1 - \frac{N}{\Omega}$ (N is the number of nucleons; for a closed shell $N = 2\Omega$ and $\chi = -1$). It is convenient to introduce the quantity

$$\theta_N = 1 - \chi_N^2 = \frac{2N}{\Omega}\left(1 - \frac{N}{2\Omega}\right).$$

Energy of quasiparticles

The energy E_ν of quasiparticles is given by the formula

$$E_\nu = \frac{1}{2}(\varepsilon'' - \varepsilon')\left[\operatorname{cth}^2\eta - 1 - 2\chi_N^2(\operatorname{cth}\eta - 1) + \frac{4\chi_N E_\nu^0}{\varepsilon'' - \varepsilon'}(\operatorname{cth}\eta - 1) + \frac{4E_\nu^{02}}{(\varepsilon'' - \varepsilon')^2}\right]^{1/2} \quad (1)$$

where $\pm E_\nu^0$ is the single particle energy ε_ν measured from the Fermi surface ($\pm$ for particles and holes, respectively) and the parameter η is given by

$$\eta = \frac{\varepsilon'' - \varepsilon'}{\Omega G}. \quad (2)$$

In the limiting case $\eta \gg 1$ (large deformations) (1) takes the form

$$E_\nu \simeq \sqrt{(\varepsilon'' - \varepsilon')^2 \theta_N e^{-2\eta} + E_\nu^{02}}. \quad (3)$$

This expression differs from the corresponding formula for a superconductor essentially only in the occupation factor θ_N. In the case $\eta < 1$ (small deformations) we get

$$E_\nu = \sqrt{\frac{1}{4}\Omega^2 G^2 + \chi_N \Omega G E_\nu^0 + E_\nu^{02}}. \quad (4)$$

(*) We give here for simplicity only the formula for uniform level density inside the shell.

In this case the energy gap in the quasiparticle spectrum does not depend on the occupation of the shell and is equal to $\frac{1}{2}\Omega G$.

The "vacuum" state corresponds to the ground state of even-even nuclei. Excited states of these nuclei are described by even numbers of quasiparticles and separated from the ground state by twice the gap. The ground state of odd-A nuclei contains one quasiparticle, which can be excited without having a gap.

Moments of inertia of strongly deformed nuclei

In the independent quasiparticle model, one gets for the moment of inertia of even-even nuclei, by employing the "cranking-model",

$$\mathfrak{J} = \mathfrak{J}' + \mathfrak{J}'' = 2\hbar^2 \sum_{\varepsilon_\nu > \lambda > \varepsilon_{\nu'}} \frac{|(j_x)_{\nu\nu'}|^2}{E_\nu + E_{\nu'}} \left(1 + \frac{E_\nu^0 E_{\nu'}^0 - \Delta^2}{E_\nu E_{\nu'}}\right)$$

$$+ \hbar^2\Delta^2 \sum_{(\varepsilon_\nu - \lambda)(\varepsilon_{\nu'} - \lambda) > 0} \frac{|(j_x)_{\nu\nu'}|^2}{E_\nu + E_{\nu'}} \cdot \frac{(\varepsilon_\nu - \varepsilon_{\nu'})^2}{E_\nu E_{\nu'}(E_\nu E_{\nu'} + E_\nu^0 E_{\nu'}^0 + \Delta^2)} \quad (5)$$

where λ is the Fermi energy, Δ is the value of the energy gap in the quasiparticle spectrum $E_\nu = \sqrt{\Delta^2 + E_\nu^{02}}$. The term $\mathfrak{J}''$ contains transitions only on one side of the Fermi surface and gives a relatively small contribution.

If we replace E_ν^0 and $E_{\nu'}^0$ in $\mathfrak{J}'$ by average values $\overline{E_\nu^0} \simeq \overline{E_{\nu'}^0} = E^0$ we get

$$\mathfrak{J}'/\mathfrak{J}_{\text{rig}} \simeq \left(1 + \frac{\Delta^2}{E^{02}}\right)^{-3/2} \quad (6)$$

where $\mathfrak{J}_{\text{rig}}$ is the moment of inertia for rigid rotation, which is obtained from (5) by setting $\Delta = 0$. It is seen that the moment of inertia is rather sensitive to the effect of the pairing correlations; thus for $\Delta \sim E^0$, which approximately corresponds to the situation in the most strongly deformed nuclei, one obtains $\mathfrak{J}'/\mathfrak{J}_{\text{rig}} \simeq \frac{1}{3}$.

The moment of inertia for odd-A nuclei contains an additional term, which arises from the odd quasiparticle in the ground state of these nuclei.

Quadrupole vibrations about spherical equilibrium shape

For nuclei in the region of closed shells the pairing interactions give rise to a spherical equilibrium shape. In order to determine the energy of the nucleus, as a function of deformation, we calculate the energy for the fixed value of the nuclear quadrupole moment Q_0. This can be done by adding to

the hamiltonian the term $-\mu Q$. Then, the lagrangian multiplier μ has to be eliminated by using the condition $\langle Q \rangle = Q_0$.

For small values of η, the nuclear potential energy has the form

$$W(\eta) = W_0 + \frac{1}{2} C_\eta \eta^2 \tag{7}$$

where the restoring force C_η is given by

$$C_\eta = \frac{1}{6} \Omega^2 G \theta_N \left(1 - \frac{\theta_N}{\theta_{N_0}} \right). \tag{8}$$

The first term gives the effect of the pairing interaction, while the second term gives the effect of the quadrupole interactions, which may be represented by a deformed field. The ratio of the two effects is characterized by the quantity θ_{N_0} representing the number of particles outside the closed shells required to make the spherical shape unstable.

For the inertial parameter of the vibrational motion one gets, by applying the cranking-model,

$$B_\eta = \frac{\hbar^2}{6} \frac{\Omega \theta_N}{(\Omega G)} \text{ (*)}. \tag{9}$$

The main contributions to B are found to arise from the change in the pairing correlations associated with a variation of η. The changes in the single particle states themselves only give a negligible contribution to B.

For the vibrational excitation energy, one gets

$$\hbar\omega = \Omega G \sqrt{1 - \frac{\theta_N}{\theta_{N_0}}} \tag{10}$$

giving a frequency which decreases as N increases. Only when N approaches the value needed to give instability, does the vibrational energy become appreciably smaller than the intrinsic excitation energy ΩG, as required by the adiabatic condition.

In order to determine the ratio of B to that corresponding to irrotational flow, we need the relation between η and the conventional deformation parameter β. Using the relation

$$Q_0 = \frac{3}{\sqrt{5\pi}} A R_0^2 \beta \tag{11}$$

(A is the atomic number, R_0 is the nuclear radius) and the connection between Q_0 and η one finds

$$B/B_{\text{irrot}} \simeq k \frac{A^{1/3}}{\Omega G \cdot N} \tag{12}$$

(*) (9) was obtained with the assumption $\eta_1 < 1$ and therefore $G > \frac{(\varepsilon'' - \varepsilon')_0}{\Omega}$ where $(\varepsilon'' - \varepsilon')_0$ is the average splitting of single particle levels in spherical nuclei caused e. g. by fluctuations of the self-consistent field.

where the numerical constant k is of order of magnitude 40-90. The values for B/B_{irr}, obtained from (12), are in qualitative agreement with the observed trends.

REFERENCES

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